



# Scalable Key Rank Estimation Algorithm for Large Keys

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Vincent Grosso

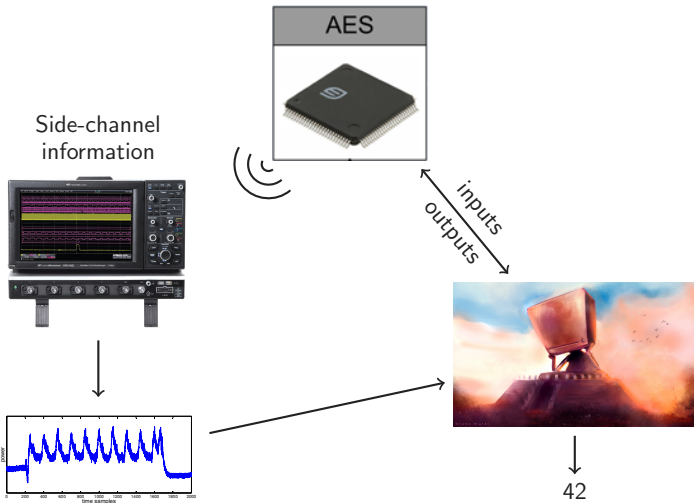
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Saint-Étienne

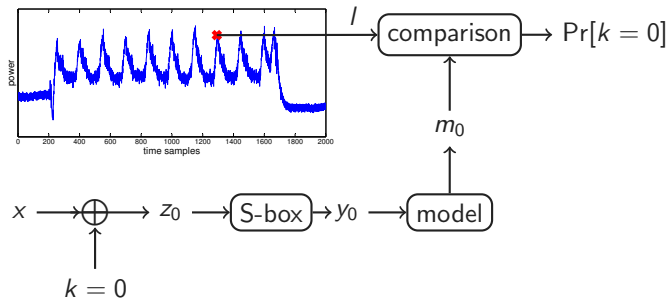
# Side-channel attacks: cryptography



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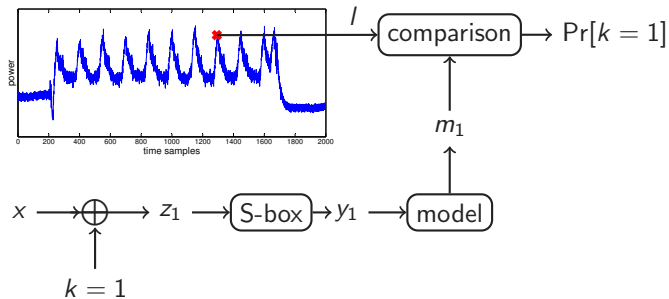


# Side-channel attacks



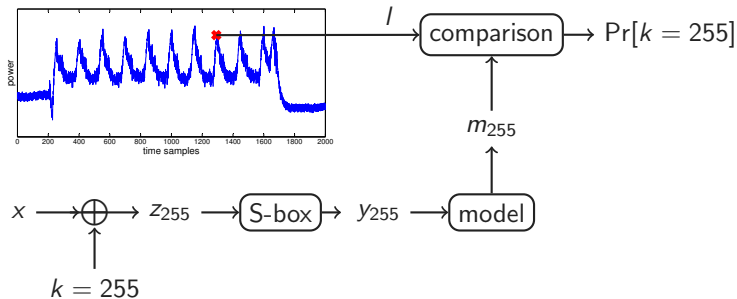
Divide-and-conquer strategy.

# Side-channel attacks



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# Side-channel attacks



Divide-and-conquer strategy.

## Side-channel attacks: result

$k_0$	$k_1$	$k_2$	...	$k_{15}$
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0Xcd,0.100	0X51,0.045	0X01,0.204		0Xdc,0.210
0Xae,0.050	0Xff,0.035	0X13,0.036		0X83,0.151
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⇒ direct recovery  
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Not Enough side-channel  
information, not enough  
computational power  
⇒ rank estimation  
(key needed, evaluation)

1. Previous solutions
2. New solution
3. Experimental results
4. Conclusion

## Previous solutions

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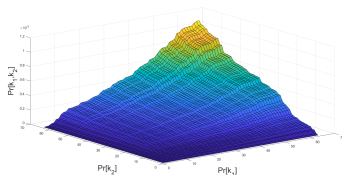
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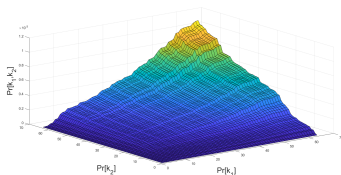
Divide-and-conquer approach on independent subkeys



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Divide-and-conquer approach on independent subkeys



Space carving  $\{k^*\}$  into 3 parts:

$$< \Pr[k|\text{SCI}] \quad ? \quad \Pr[k|\text{SCI}] <$$

Smaller is the part  $?$  the more accurate is the rank estimation



## Rank estimation algorithms zoo

Method	Pros	Cons
Eurocrypt'13	First solution, can compute the exact rank (in theory)	Quite slow, quite loose bounds (in practice)
Pro'15		
FSE'15		
Asiacrypt' 15		
CT-RSA'17		
CHES'17		
This paper		

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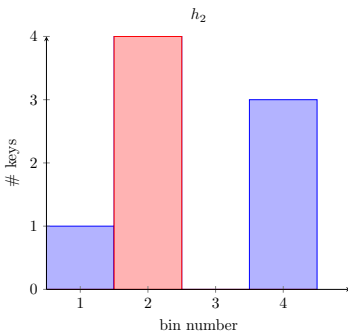
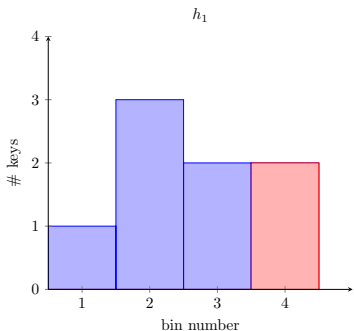
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This paper	Fast, tight estimation of the rank even for large key	Less efficient than CHES'17

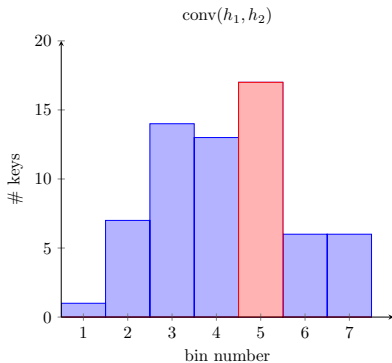
# The histogram solution (FSE'15)

Candidate	Pr	$k_1$		bin	$k_2$		
		log	bin		Pr	log	bin
0	0.6643	-0.5901	1	0.0012	-9.7027	3	
1	0.2588	-1.9501	1	0.0011	-9.8283	3	
2	0.0313	-4.9977	2	0.3588	-1.4787	1	
3	0.0412	-4.6012	2	0.0713	-3.8100	1	
4	0.0001	-13.2877	4	0.5643	-0.8255	1	
5	0.0020	-8.9658	3	0.0012	-9.7027	3	
6	0.0013	-9.5873	3	0.00005	-14.2877	4	
7	0.0010	-9.9658	3	0.00205	-8.9302	3	

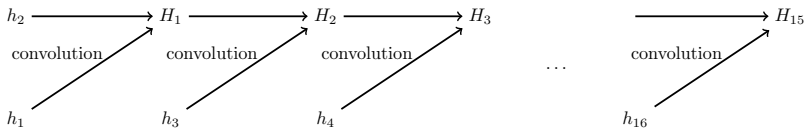


Perform convolution of histogram

$$\text{conv}(h_1, h_2)[i] = \sum_{j=0}^i h_1[j]h_2[i-j]$$



# Mix more results

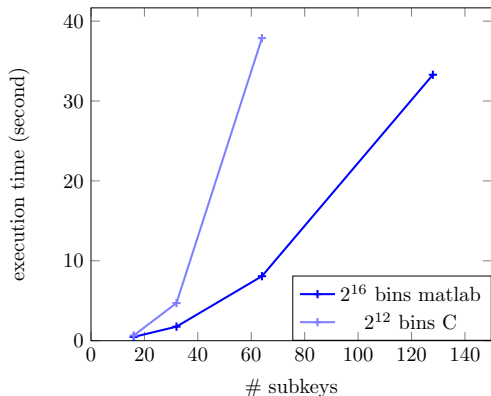


Size of  $H_i$  grows



## Limitation for larger keys

For large number of dimension we perform convolution on larger and larger histograms: could be costly.

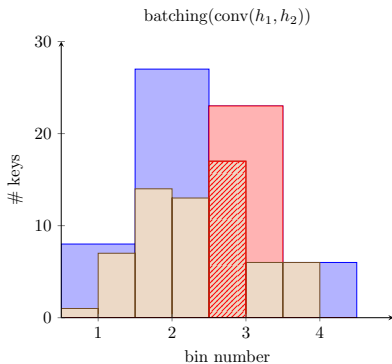


Can the cost be linear in the number of subkeys?

**New solution**

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Keep the size of the histogram constant

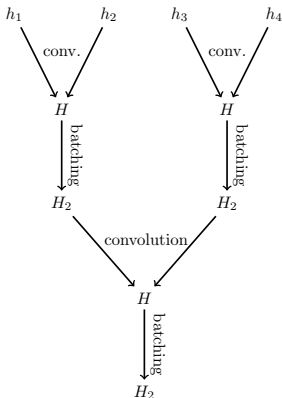


## Why does it work?

Similar to classical histogram solution we can keep track of the position (bin of the key)

Similar as dividing the number of bins by the number of subkeys

Convolution need equally bin sized histogram: need a balanced tree structure



## Why is more efficient?

Similar as doing classical histogram convolution with large bin

But a better tracking of the estimation error

$\nu$  : number of subkeys

$\epsilon$ : size of bin /2

Method	error	cost
Classical FSE'15	$\nu\epsilon$	quadratic
Reduced FSE'15	$\nu^2\epsilon$	linear
Batching	$(\nu + \log_2(\nu)\frac{\nu}{2})\epsilon$	linear

## Experimental results

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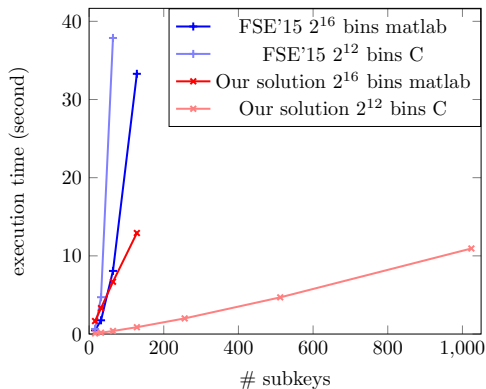
- ▶ Matlab implementation (limited to 1024-bit key)
- ▶ C implementation

Leakages: subkey+noise (S-box)

Size of subkey: 8-bit

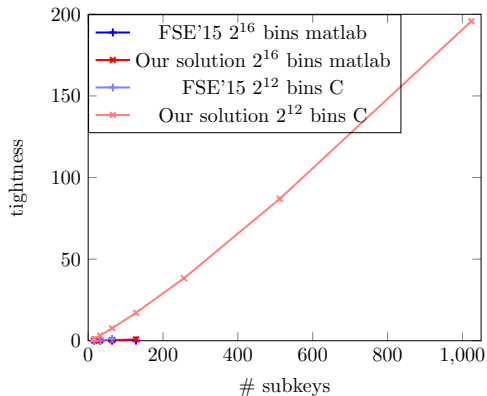
Number of subkeys: 8-1024

Number of bins: tightness-efficiency parameter

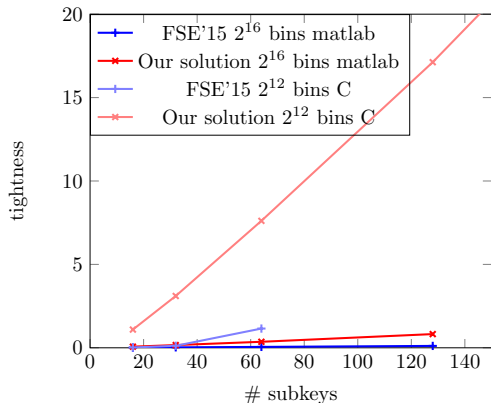


Our solution has a complexity linear in the number of subkeys

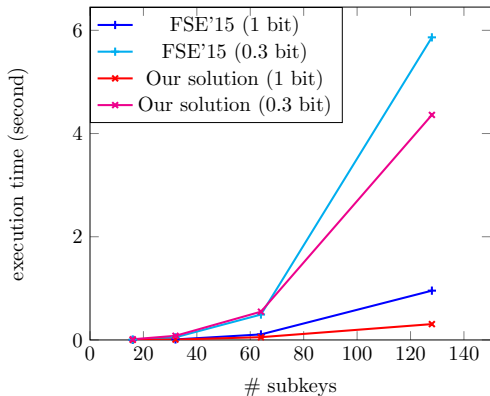




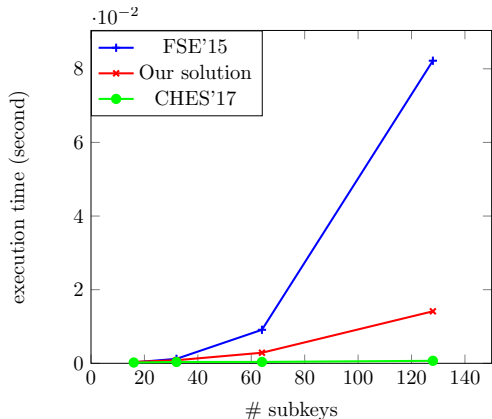
Works for very long key, with tightness damage



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Adapted number of bin to have similar tightness



CHES'17 solution is not so tight (6 bits, and cannot be tightened) all solutions are efficient ( $< 0.1s$ )

## Conclusion

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Trick for rank estimation for large keys

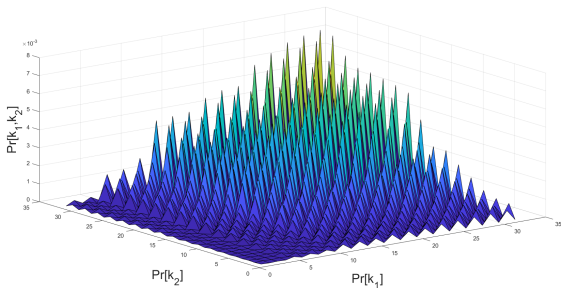
Tight and efficient method

# Conclusion & open problems

Trick for rank estimation for large keys

Tight and efficient method

Limited to independent attack



Thanks!

Questions?

Comments?